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Constraints on primordial black holes as dark matter candidates from capture by neutron stars

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We put constraints on primordial black holes (PBHs) as dark matter candidates by considering their capture by neutron stars (NSs). If a PBH is captured by a NS, the latter is accreted onto the PBH in a very short time leading to the star's destruction. Mere observations of NSs put limits on the abundances of PBHs. We are able to exclude PBHs as dark matter from observations of NSs in globular clusters for PBHs masses $10^{18}\text{g} \lesssim m_{\rm BH} \lesssim 5 \times 10^{23}\text{g}$.

I. INTRODUCTION

The existence of the dark matter (DM) has been established so far only through its gravitational interaction. Consequently, little is known about the DM nature apart from the fact that it is non-baryonic, non-relativistic, weakly interacting and constitutes about 23% of the total energy budget of the Universe (for a recent review see, e.g., [1, 2]).

Various candidates for the DM have been considered in the literature. In the context of particle physics they are associated with a new stable particle beyond the Standard Model, a popular example being the so-called Weakly Interacting Massive Particles (WIMPs). However, candidates that do not require new stable particles also exist and are still viable. An attractive candidate of this type is primordial black holes (PBHs) [3, 4]. This is the possibility we consider in this paper.

In the early universe, some primordial density fluctuations could have collapsed producing a certain amount of black holes. These PBHs possess properties that make them viable DM candidates: they are nonrelativistic and have a microscopic size of the order $r \sim 10^{-8}\text{cm}\,(m_{\rm BH}/10^{20}\text{g})$, which makes them effectively collisionless. The initial mass function of PBHs depends on their production mechanism in the early universe and is, essentially, arbitrary.

There exist a number of observational constraints on the fraction of PBHs in the total amount of DM. First, PBHs with masses $m_{\rm BH} \leq 5 \times 10^{14}\text{g}$ evaporate due to Hawking radiation [5] in a time shorter than the age of the Universe and cannot survive until today. At slightly larger masses, even though the PBH lifetime is long enough, the Hawking evaporation still poses a problem: the PBHs emit γ-rays with energies around 100MeV [6] in the amount that contradicts the data on the extragalactic gamma-ray background. For instance, the Energetic Gamma Ray Experiment Telescope [7] has put an upper limit on the cosmological density $\Omega_{\rm PBH} \leq 10^{-9}$ for $m_{\rm BH} = 10^{15}\text{g}$ [8]. From such observations, one can infer that PBHs with masses $m_{\rm BH} \leq 10^{16}\text{g}$ cannot constitute more than 1% of the DM. More massive PBHs were constrained by EROS microlensing survey and the MACHO collaboration, which set an upper limit of 3% on the fraction of PBHs in the Galactic halo in the mass range $10^{26}\text{g} < m_{\rm BH} < 10^{30}\text{g}$ [9, 10]. Theoretical predictions on the abundance of PBHs have also been computed using microlensing of Kepler stars [11, 12]. At even larger masses $10^{33}\text{g} < m_{\rm BH} < 10^{40}\text{g}$, the three-year Wilkinson Microwave Anisotropy Probe (WMAP3) data and the COBE Far Infrared Absolute Spectrophotometer (FIRAS) data have been used to put limits on the abundance of PBHs [13]. These constraints are shown in Fig. 1. They leave open the window of masses (a few) $\times\, 10^{16}\text{g} < m_{\rm BH} < 10^{26}\text{g}$.

In order to put constraints on PBHs in the remaining allowed mass range, in Ref. [14] we have considered the capture of PBHs by a star during star formation process and their further inheritance by the star's compact remnant, the neutron star (NS) or the white dwarf (WD). The presence of even a single PBH of a corresponding mass inside the remnant (NS or WD) leads to a rapid destruction of the latter by the accretion of the star matter onto the PBH [15–19]. Thus, mere observations of NSs and WDs in a DM-rich environment, such as centers of globular clusters, impose constraints on the fraction of PBHs in the DM and exclude PBHs as the only DM candidate in the range of masses $10^{16}\text{g} < m_{\rm BH} < 3 \times 10^{22}\text{g}$. Still, a range of PBHs masses from $3 \times 10^{22}\text{g}$ to 10^{26}g remains unconstrained.

In this paper we derive the constraints that arise from the direct capture of PBHs by already existing NSs. The origin of the constraints is the same as in Ref. [14]: even a single PBH captured by a compact star rapidly destroys the latter, so the existing observations of the NSs and

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WDs require that the probability of capture is much less than one. Assuming that, we put constraints on the PBH abundance.

We find that the argument based on the capture of PBHs by the existing NSs allows one to extend the constraints to larger PBH masses and exclude PBHs as comprising 100% of the DM up to $m_{\rm BH} \lesssim 5 \times 10^{23}$ g, leaving open only a small window of about two orders of magnitude. Also, the constraints on the fraction $\Omega_{\rm PBH}/\Omega_{\rm DM}$ of PBHs in the total amount of DM at large PBH masses become tighter as compared to Ref. [14]. The final situation is summarized in Fig. 1.

The rest of this paper is organized as follows. In Sect. II we discuss the capture of PBHs by compact remnants due to dynamical friction. In Section III we derive the constraints on the fraction of PBHs in the DM from the observations of NSs. In Section IV we summarize the results and present our conclusions. Throughout the paper, we use the units $\hbar = c = 1$

II. CAPTURE OF BLACK HOLES BY COMPACT STARS

A. Energy Loss

A PBH is captured if, during its passage through a star, it loses its initial energy and becomes gravitationally bound. From this moment every subsequent PBH orbit will again pass through the star, so that finally the PBH will lose all its energy and will remain inside the star all the time. Therefore, the criterion of capture of a PBH is $E_{\rm loss} > m_{\rm BH} v_0^2/2$ with $E_{\rm loss}$ being the energy loss during the collision and v_0 the PBH asymptotic velocity. Two mechanisms of energy loss are operating during the collision: deceleration of the PBH due to the accretion of star's material and the so-called dynamical friction [20, 21]. In the relevant mass range of PBHs the former mechanism is less efficient compared to the latter. We, therefore, focus here on the energy loss due to the dynamical friction.

As a PBH passes through the star, it transfers momentum and energy to the surrounding matter. The result, called the dynamical friction, is a net force that is opposite to the direction of motion of the PBH. As long as the PBH velocity v during the collision is larger than the velocity of the particles constituting the compact object (which is a good approximation for compact stars), one may take the dynamical friction force to be

$$\mathbf{f}_{\rm dyn} = -4\pi G^2 m_{\rm BH}^2 \rho \ln\Lambda \frac{\mathbf{v}}{v^3}, \tag{1}$$

where ρ is the density of the star matter. For the sake of the estimate, it can approximately be considered as constant, $\rho = 3M/(4\pi R^3)$, with M and R being the mass and the radius of the compact star. The factor $\ln(\Lambda)$ is the so-called Coulomb logarithm [20, 21].

The total BH energy loss can easily be computed from eq. (1). Taking the PBHs velocity during the collision to be $v = v_{\rm esc} = \sqrt{2GM/R} \gg v_0$, the energy loss during one collision is

$$E_{\rm loss} = \frac{3Gm_{\rm BH}^2 \ln\Lambda}{R}. \tag{2}$$

As one can see, the energy loss depends on two parameters: the PBHs mass and the radius of the compact object. Since $E_{\rm loss}$ is inversely proportional to the radius of the star, NSs induce a larger energy loss during one collision compared with WDs. Thus, we will only consider the case of NSs from now on. In this case

$$E_{\rm loss}/m_{\rm BH} = 6.7 \times 10^{-11} \left(\frac{m_{\rm BH}}{10^{22}\rm g}\right), \tag{3}$$

where we have substituted $R = 10$ km.

A remark is in order. The core of a neutron star is comprised of the degenerate neutron gas, so the question arises whether eq. 1 is still applicable[1]. Note, however, that by the time the falling PBH reaches the star it picks a relativistic velocity $v \sim 0.6c$ and can transfer to neutrons the momentum of up to ~ 1.8 GeV, which is by a factor of a few larger than the Fermi momentum of neutrons in the center of the star, and much larger than the Fermi momentum away from the center. So, while some reduction as compared to the estimate (3) is expected, this reduction is not dramatic and is comparable in magnitude with other uncertainties involved in the problem. We do not attempt here to calculate the reduction factor (which is complicated both technically and conceptually). Instead, we will present the final constraints with the uncertainty band corresponding to energy losses as given by eq. (2) and reduced by a factor of 5.

It remains to be checked that, once the PBH becomes gravitationally bound, multiple collisions rapidly bring the PBH inside the NS. Assuming a radial orbit and denoting the apastron $r_{\rm max}$, the half-period is

$$\Delta T = \frac{\pi r_{\rm max}^{3/2}}{\sqrt{GM}}.$$

The energy loss as a function of $r_{\rm max}$ follows from eqs. (1) and (2),

$$\Delta E = -\frac{3Gm_{\rm BH}^2 r_{\rm max} \ln\Lambda}{R(r_{\rm max} - R)},$$

where R is the star radius. Dividing the energy loss by the time and expressing the energy in terms of $r_{\rm max}$ one obtains the differential equation for the evolution of $r_{\rm max}$ as a function of time,

$$\dot{\xi} = -\frac{1}{\tau}\frac{\xi^{3/2}}{\xi - 1}, \tag{4}$$

[1] We thank A. Gould for raising this issue.

where $\xi = r_{\max}/R$ and

$$\tau = \frac{\pi M R^{3/2}}{3m_{\rm BH}\sqrt{GM}\ln\Lambda} \simeq 6.1\times 10^5{\rm s}\left(\frac{m_{\rm BH}}{10^{22}{\rm g}}\right)^{-1}.$$

Assuming $\xi \gg 1$, which is correct for the most part of the process, and solving eq. (4) the energy loss time is

$$t_{\rm loss} \simeq 2\tau\sqrt{\xi_0},$$

where the initial value ξ_0 can be estimated as $\xi_0 \sim M/(3m_{\rm BH}\ln\Lambda)$, by requiring the initial PBH energy to be of the order of $E_{\rm loss}$. Assembling all the factors one has

$$t_{\rm loss} \simeq 1.8\times 10^3{\rm yr}\left(\frac{m_{\rm BH}}{10^{22}{\rm g}}\right)^{-3/2}. \qquad (5)$$

Thus, PBHs heavier than $m_{\rm PBH} \gtrsim 3\times 10^{17}$ g end up inside the NS in a time shorter than 10^{10} yr.

B. Capture Rate

In order to calculate the capture rate, we assume that the PBHs follow a Maxwellian distribution in velocities with the dispersion $\bar{v}$,

$$dn = n_{\rm BH}\left(\frac{3}{2\pi\bar{v}^2}\right)^{3/2}\exp\left\{\frac{-3v^2}{2\bar{v}^2}\right\}d^3v, \qquad (6)$$

where $n_{\rm BH} = \rho_{\rm BH}/m_{\rm BH}$, $\rho_{\rm BH}$ being the density of PBHs at the star location. It can be expressed in terms of the local DM density $\rho_{\rm DM}$ as follows,

$$\rho_{\rm BH} = \frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}}\rho_{\rm DM}. \qquad (7)$$

Following [16, 22] where relativistic corrections have been neglected, the capture rate takes the form

$$F = \frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}}F_0, \qquad (8)$$

where

$$F_0 = 2\sqrt{6\pi}\frac{\rho_{\rm DM}}{m_{\rm BH}}\frac{GMR}{\bar{v}}\left(1-\exp\left(-\frac{3E_{\rm loss}}{m_{\rm BH}\bar{v}^2}\right)\right) \qquad (9)$$

is the capture rate assuming PBHs comprise all of the DM, and $E_{\rm loss}$ is given by eq.(2).

Two different regimes are possible. In the case when the energy loss is small, $E_{\rm loss} \ll m_{\rm BH}\bar{v}^2/3$, the exponential can be expanded and one gets at the leading order

$$F_0 = 6\sqrt{6\pi}\frac{\rho_{\rm DM}}{m_{\rm BH}}\frac{GMR}{\bar{v}^3}\frac{E_{\rm loss}}{m_{\rm BH}}. \qquad (10)$$

In view of eq. (2) the capture rate is independent of $m_{\rm BH}$ in this regime. In the opposite case $E_{\rm loss} \gg m_{\rm BH}\bar{v}^2/3$ the exponential in eq. (9) can be neglected and

$$F_0 = 2\sqrt{6\pi}\frac{\rho_{\rm DM}}{m_{\rm BH}}\frac{GMR}{\bar{v}}, \qquad (11)$$

so that the capture rate decreases with increasing $m_{\rm BH}$. In both cases the capture rate is inversely proportional to some power of velocity and is thus maximum for sites with high dark matter density $\rho_{\rm DM}$ where the velocity dispersion $\bar{v}$ is small.

III. CONSTRAINTS

As previously mentioned, if a NS captures a PBH, the accretion of the NS material onto the PBH rapidly destroys the star. Therefore, observations of NSs imply constraints on the capture rate of PBHs which has to be such that the probability of the PBH capture is much less than one. In view of eq. (8) these constraints translate into constraints on the fraction of PBHs in the dark matter, $\Omega_{\rm PBH}/\Omega_{\rm DM}$.

Given a NS of the age $t_{\rm NS}$, the probability of its survival is $\exp(-t_{\rm NS}F)$ with F given by eqs. (8) and (9). Requiring that the survival probability is not small leads to the constraint

$$\frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}} \leq \frac{1}{t_{\rm NS}F_0}. \qquad (12)$$

Depending on the environment where the NS is located, F_0 may vary by many orders of magnitude. The most stringent constraints come from sites where F_0 is high. Among such sites, Globular Clusters (GCs) provide the best constraints.

GCs are compact, nearly spherical collections of stars scattered over the Galactic halo. They have ages between 8 to 13.5 Gyr, and as such are the oldest substructures of our Galaxy. GCs are made of population II stars, WDs, NSs and black holes. A typical GC has an average radius of 30 pc, a core radius of 1 pc and a baryonic mass of (a few) $\times\, 10^5 M_\odot$ [23].

The DM content of GCs is a matter of an ongoing debate. The distribution of metallicity in GCs is bimodal, indicating two subpopulations formed by different mechanisms [24]. The metal-rich GCs are considered to be formed during major gas mergers in protogalaxies [25–28]. These GCs contain very little DM, if any. Instead, the metal-poor GCs are thought to have been formed in low-mass dark matter halos at very high-redshift $z \sim 10-15$ [29–34]. Observations of GCs show no evidence of DM halos [35]. This is expected as the halos should have been tidally stripped due to interactions with the Galaxy [36]. The DM content would, however, be preserved in the cores of such GCs. In support of this picture, it has been found in Refs. [31, 36], using high-resolution N-body simulations, that many properties of simulated GCs with DM halos are similar to those of observed GCs. In what follows we will focus on metal-poor GCs and assume that they have been formed in DM halos and thus possess DM-rich cores.

In Ref. [37] the DM density close to the GC core has been estimated to be of the order $\rho_{\rm DM} \sim 2\times 10^3{\rm GeV\,cm^{-3}}$. Such a DM density was concluded to be

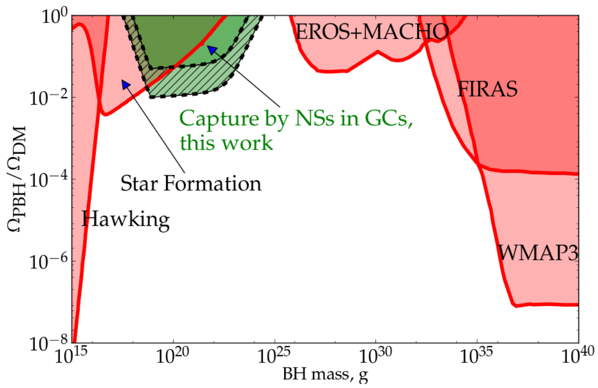

FIG. 1. Constraints on the fraction of PBHs in the total amount of DM. Shaded regions are excluded. PBHs with masses $m_{\rm BH} \lesssim 10^{16}$g would have evaporated by now or overproduced γ-rays. EROS microlensing survey puts limits on the contribution to DM consisting of PBHs in the mass range 10^{26} g $< m_{\rm BH} < 3 \times 10^{34}$ g from microlensing. In [14] the amount of DM adiabatically contracted during star formation constrains PBHs in the mass range 10^{16} g $< m_{\rm BH} <$ 3×10^{22} g. The analysis of [13] constrains the mass range 10^{33} g $< m_{\rm BH} < 10^{40}$ g from measurements of the cosmic microwave background by FIRAS and WMAP3. The green shaded region is excluded by capture of PBHs by NSs in GCs. The green hatched region corresponds to constraints that can be reached assuming a less conservative choice of parameters and a better efficiency of the dynamical friction.

rather independent of the original halo mass and is in agreement with N-body simulations [31, 36]. Therefore, we adopt this value in our estimates.

The velocity dispersion is another important parameter. Since stars are collisionless and therefore behave similarly to DM particles, this parameter can be extracted from observations. We adopt the value of $\bar{v} = 7$ km s^{-1} (the velocity dispersion varies noticeably from cluster to cluster; for individual values see Ref.[38]). Finally, we adopt the NS radius $R_{\rm NS} = 10$ km, the mass $M_{\rm NS} =$ $1\ M_\odot$, and the life time $t_{\rm NS} = 10^{10}$yr [39].

The constraints coming from observations of NSs in GCs, as well as previously existing constraints are summarized in Fig. 1. As mentioned earlier, the lack of knowledge of the efficiency of the dynamical friction in case of NS is reflected in the uncertainty band. As one can see, the most conservative constraints exclude the PBHs as the unique DM component for masses lower than $m_{\rm BH} \sim 5 \times 10^{23}$g, thus pushing up the previously existing threshold by more than an order of magnitude.

In qualitative terms, the shape of the exclusion region in Fig. 1 is easy to understand from eqs. (10) and (11). The horizontal part of the curves is due to eq. (10) where the dependence on the PBH mass cancels out (cf. eq. (2)). The inclined part on the right results from eq. (11). The transition between the two regimes occurs at the PBH mass such that $E_{\rm loss} \sim m_{\rm BH}\bar{v}^2/3$. The left part of the constraints curves comes from the time needed for multiple collisions to bring the PBH inside the NS.

IV. CONCLUSIONS

We have derived constraints on the fraction of PBHs in the total amount of DM from observations of NSs in globular clusters. These constraints arise from the requirement that PBHs are captured by NSs with probability much less than one, since capture of even a single PBH leads to a rapid accretion of the star matter onto the PBH and eventual star destruction.

The most conservative constraints exclude PBH as the only DM candidate for masses 10^{18}g $\le m_{\rm BH} \le 5 \times 10^{23}$g, but accounting for uncertainties can result into the exclusion of masses 3×10^{17}g $\le m_{\rm BH} \le 2 \times 10^{24}$g. Taking into account the previously existing constraints, only a small window of masses around 10^{25} g remains where PBHs can still constitute all of the DM. Note however that this would require a PBH production scenario capable of creating a very narrow PBH mass distribution function. Those scenarios where the PBH mass function is wider than about two orders of magnitude are altogether excluded.

The present constraints are complementary to the constraints derived in Ref. [14]. An extra window of masses 3×10^{22}g $\le m_{\rm BH} \le 5 \times 10^{23}$g has been constrained at the percent level. In Ref. [14] better constraints are achieved for masses 10^{16}g $\le m_{\rm BH} \le 8 \times 10^{19}$g, while the present paper reaches more competitive constraints for masses $m_{\rm BH} \ge 8 \times 10^{19}$g. It is also important to note that different assumptions are required in these cases: while the constraints of Ref. [14] are sensitive to the DM distribution at the epoch of the GC formation, for the constraints derived in this paper the present-epoch DM distribution in GCs is relevant.

We did not present the constraints that come from observations of the Galactic center, which is another relatively close region of high DM density. If the DM density in the Galactic center is comparable to that assumed above for the the cores of the GCs, no new constraints arise from that region [40]. This is because the capture rate depends strongly on the PBH velocity dispersion, cf. eq. (10), which is by more than an order of magnitude larger in the Galactic center than in the cores of GCs. It has been suggested, however, that the DM density in the Galactic center may be as high as $\rho_{\rm DM} = 10^6$GeV cm^{-3} [41]. If this were confirmed, the constraints from the Galactic center would close the remaining allowed PBH mass window.

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